

# CS 4810: Homework 9

due 11/26 11:59pm

(your name + netid)

Collaborators: (names and netids)

Each problem is worth 33 points.

## Problem 1

Construct a family of directed graphs such that the random-walk algorithm takes exponential time in expectation to solve the PATH problem on these instances.

## Problem 2

A directed graph is called *strongly connected* if for every pair of vertices, there exists a directed paths between them (in both directions).

Show that the PATH problem reduces in logarithmic space to the problem of deciding strong connectivity.

## Problem 3

Show that there exists a polynomial-space algorithm for the following problem (from the previous homework)

$$L = \{\varphi \mid \forall x. \exists y. \varphi(x, y) = 1\}.$$

(Here,  $\varphi$  is a Boolean formula in variables  $x_1, x_2, \dots$  and  $y_1, y_2, \dots$ )