CS 4810: Homework 8

due 11/19 11:59pm

(your name + netid)

Collaborators: (names and netids)

Each problem is worth 25 points.

Problem 1

Show $\mathbf{P} = \mathbf{NP}$ would imply that there exists a polynomial-time algorithm for the following problem: Given a Boolean formula φ in variables $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$, decide the statement $\forall x. \exists y. \varphi(x, y)$.

Problem 2

Show that $\Pr_{x \in \{0,1\}^n}\{x_1 + \cdots + x_n = 0 \bmod 2\} = 1/2$.

Problem 3

Let $\mathbf{RP}$ be the set of languages L such that there exists a randomized algorithm M with these properties:

- For every $x \in L$, the algorithm M on input x always accepts (it accepts with probability 1).
- For every $x \notin L$, the algorithm M on input x accepts with probability at most 1/2.
- The algorithm M runs in polynomial time (it halts after a polynomial steps with probability 1).

Let L be a language with $L \in \mathbf{RP}$ and $\neg L \in \mathbf{RP}$ show that there exists a randomized algorithm M with these properties

- For every string x, the expected running time of M on input x is polynomial.
- The error probability of the algorithm M is 0 for every possible input x. (The algorithm on input x accepts if and only $x \in L$.)

Problem 4

Let L be a language. Suppose there exist numbers s and c with $0 < s < c < 1$ and a randomized algorithm M with these properties:

- For every $x \in L$, the algorithm M on input x accepts with probability at least c.
- For every $x \notin L$, the algorithm M on input x accepts with probability at most s.

Show that $L \in \mathbf{BPP}$.

Hint: You can use the inequality $\Pr\{|X - \mu_X| \geq t \cdot \sigma_X\} \leq 1/t^2$, where $\mu_X = \mathbb{E}X$ is the mean of X and $\sigma_X = (\mathbb{E}(X - \mu)^2)^{1/2}$ is the standard deviation of X. (This inequality is often called Chebyschev's inequality.)