CS 4810: Homework 7

due 10/31 11:59pm

(your name + netid)

Collaborators: (names and netids)

Each problem is worth 25 points.

Problem 1

Show that **NP** is closed under union, concatenation, intersection, and Kleene star.

Problem 2

Show that the following language is **NP**-complete (polynomial-time reduction from SAT),

$\text{DOUBLE SAT} = \{\langle \varphi \rangle \mid \varphi \text{ is a Boolean formula with at least two satisfying assignments}\}$.

Problem 3

Let G be a graph. A *clique* in G is a subset C of vertices such that any two vertices $u, v \in C$ are adjacent. A *vertex cover* of G is a subset S of vertices such that every edge of G has at least one endpoint in S.

Show polynomial-time reductions between the following languages,

$$\text{CLIQUE} = \{\langle G, k \rangle \mid G \text{ contains a clique of size } k\},$$
$$\text{VERTEX COVER} = \{\langle G, \ell \rangle \mid G \text{ has a vertex cover of size } \ell\}.$$

Problem 4

This problem investigates *resolution*, a method for proving the unsatisfiability of CNF formulas. Let $\varphi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a CNF formula with clauses $C_1, \ldots, C_m$. Let

$$\mathcal{C} = \{C_i \mid C_i \text{ is a clause of } \varphi\}.$$

A resolution step proceeds as follows: Take two clauses C_a and C_b in $\mathcal{C}$, which both have some variable x occurring positively in one of the clauses and negatively in the other. Thus, $C_a = (x \vee y_1 \vee y_2 \vee \cdots \vee y_k)$ and $C_b = (\neg x \vee z_1 \vee z_2 \vee \cdots \vee z_\ell)$, where the y_i and the z_j are literals. Form the new clause $(y_1 \vee y_2 \vee \cdots \vee y_k \vee z_1 \vee z_2 \vee \cdots \vee z_\ell)$ and remove repeated literals. Add this new clause to $\mathcal{C}$.

Repeat the resolution steps until no additional clauses can be obtained. If the empty clause () is in $\mathcal{C}$, then declare φ unsatisfiable.

Say that resolution is sound if it never declares satisfiable formulas to be unsatisfiable. Say that resolution is complete if all unsatisfiable formulas are declared to be unsatisfiable.

Part a

Show that resolution is sound and complete.

Part b

Use Part a to show that 2-CNF SAT $\in$ **P**. (2-CNF SAT is the set of satisfiable formulas in conjunctive normal form with at most two literals per clause.)