CS 4810: Homework 3

due 09/19 11:59pm

(your name + netid)

Collaborators: (names and netids)

Each problem is worth 20 points.

Problem 1

Let M be a non-deterministic finite automaton. Show that there exists a non-deterministic finite automaton M' with the same language as M, the same number of states as M, but without any ε-transitions.

Problem 2

For a word $w = x_1 \dots x_n \in \Sigma^*$, define the *reverse* of w, w^R, to be $x_n \cdots x_1$.

Let $L \subseteq \Sigma^*$ be a regular language.

Show that the reverse language $L^R := \{w^R \mid w \in L\}$ is also regular.

Problem 3

Let $L \subseteq \Sigma^*$ be a regular language.

Show that the cycle language,

$$\mathrm{cycle}(L) = \{w_2 w_1 \mid w_1 w_2 \in L\},$$

is also regular.

Problem 4

Let $L \subseteq \Sigma^*$ be some language over Σ, not necessarily regular. Define an equivalence relation $\sim_L$ on Σ^*, the set of finite strings over Σ, by saying that $u \sim_L v$ if and only if for any $w \in \Sigma^*$, either both uw and vw are in L or neither is.

(a) Suppose that L_M is the language accepted by some DFA M. Show that the number of $\sim_{L_M}$-equivalence classes is finite. (Hint: Consider the state that M will be in after reading a given string u.)

(b) Consider the language $L_{\text{match}} = \{a^n b^n \mid n \geq 0\}$. Show that there are infinitely many $\sim_{L_{\text{match}}}$-equivalence classes.

(c) Show that if for some language L, there are only finitely many $\sim_L$-equivalence classes, then L is regular.

Problem 5

(a) For languages $A, B \subseteq \Sigma^*$, the *perfect shuffle* of A and B is the language

$$\{x_1 y_1 \cdots x_k y_k \quad | \quad x_1 \cdots x_k \in A \text{ and } y_1 \cdots y_k \in B, \\ \text{symbols } x_1, \ldots, x_k, y_1, \ldots, y_k \in \Sigma\}$$

Show that if A and B are regular, then the perfect shuffle of A and B is also regular.

(b) For languages $A, B \subseteq \Sigma^*$, the *shuffle* of A and B is the language

$$\{u_1 v_1 \cdots u_k v_k \quad | \quad u_1 \cdots u_k \in A \text{ and } v_1 \cdots v_k \in B, \\ \text{strings } u_1, \ldots, u_k, v_1, \ldots, v_k \in \Sigma^*\}$$

Show that if A and B are regular, then the shuffle of A and B is also regular.