CS 6810 – Theory of Computing

Lecture 1: Introduction & Review

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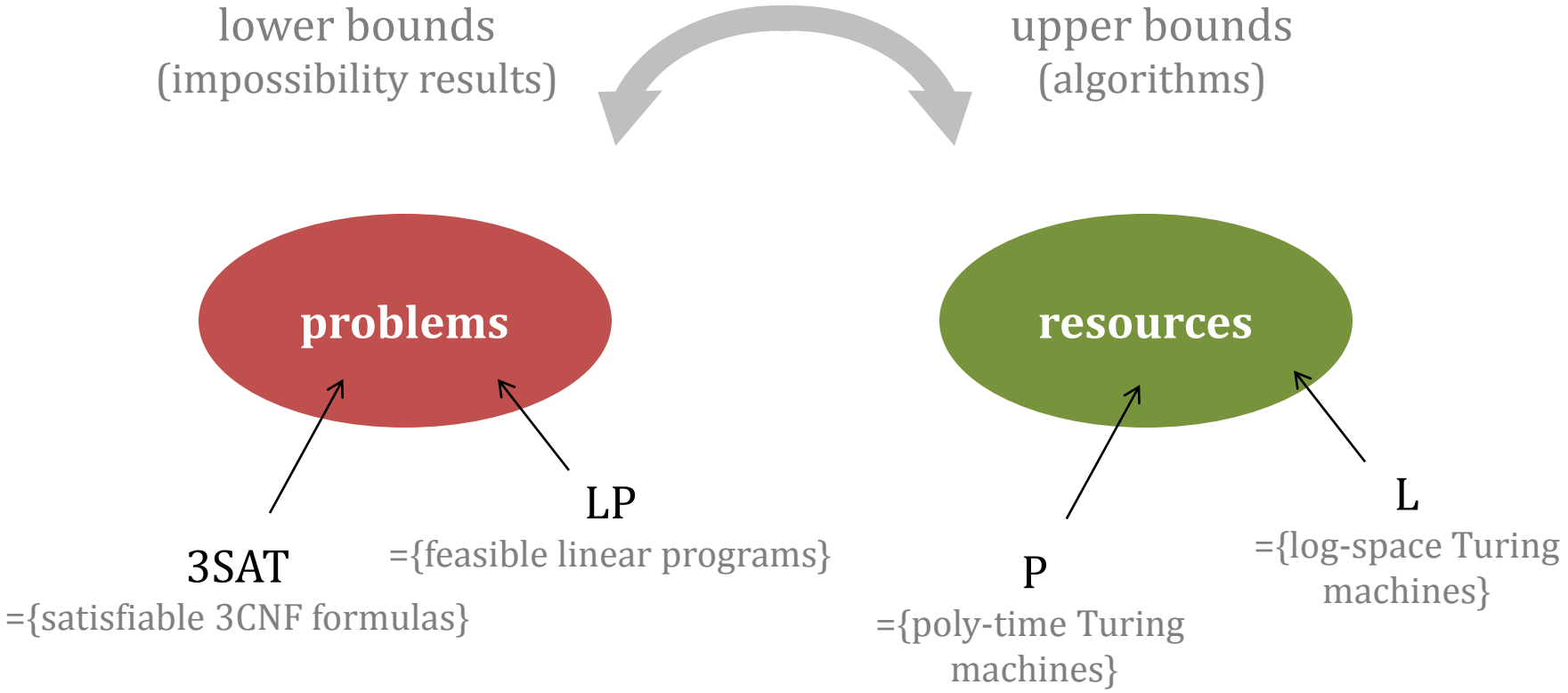
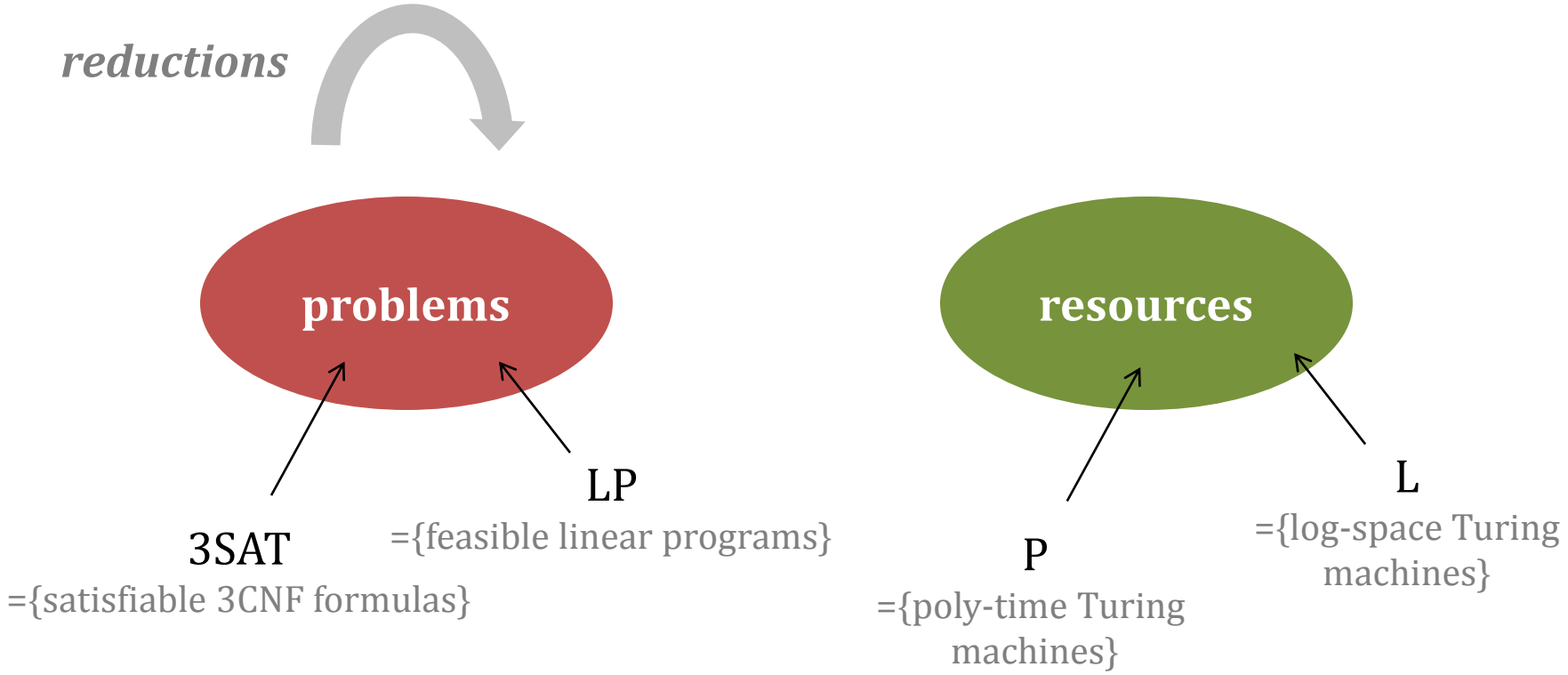
What is complexity theory?

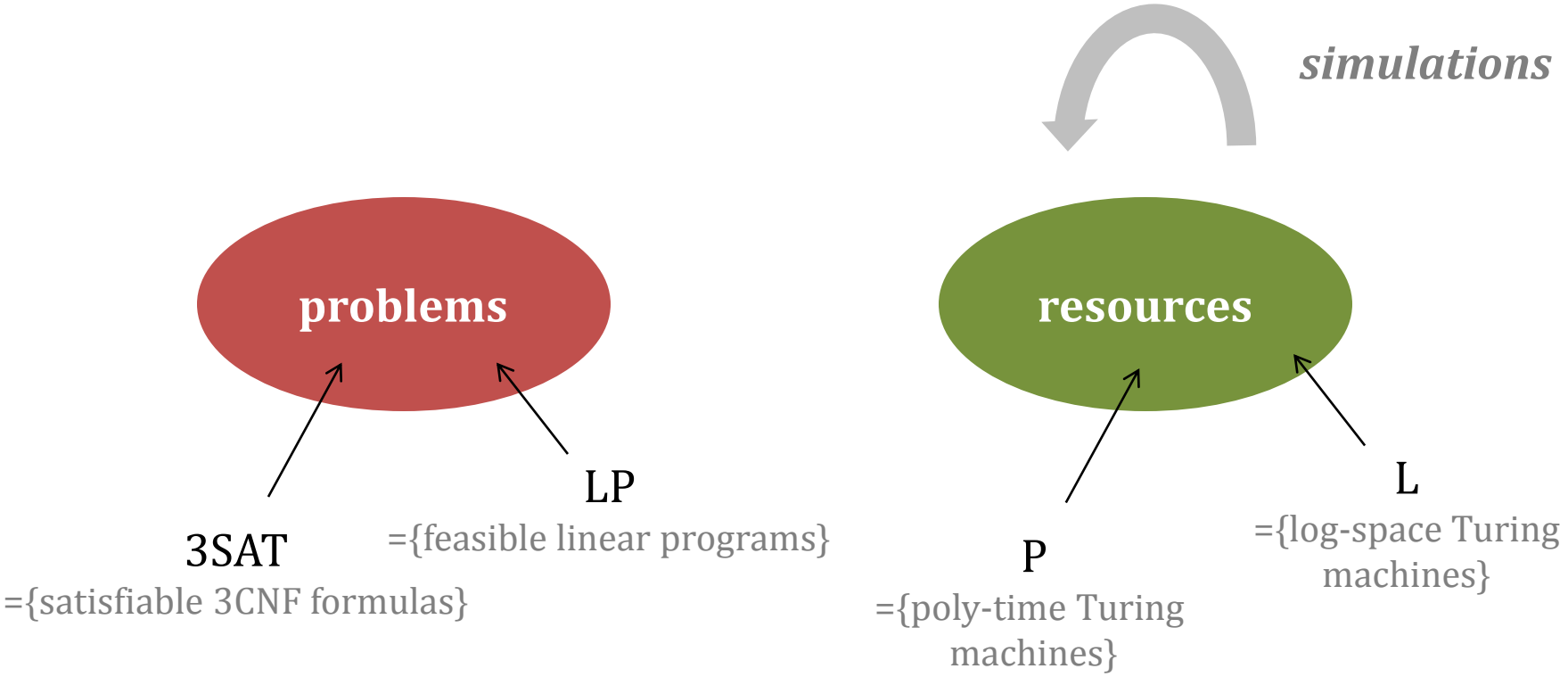
How do resource limitations impact our ability to solve problems?



Which problems are easier than others?



What resources are more powerful than others?



decision problems *(simplest, default)*

search problems *(most general)*

promise problems *(very useful case between decision and search)*

distributional problems *(average-case complexity, later in course)*

decision problems *(simplest, default)*

$f: \{0,1\}^* \rightarrow \{0,1\}$ or $L \subseteq \{0,1\}^*$

(YES/NO answer for each input) (set of YES inputs)

Example

3SAT = {satisfiable 3CNF formulas ϕ}

search problems *(most general)*

for every input x, there is a set of acceptable outputs y

relation $R = \{(x, y) |$ output y is acceptable on input $x\}$

Examples

$3SAT^s = \{(\phi, z) \mid$ assignment z satisfies 3CNF formula $\phi\}$

$Max3SAT^s = \{(\phi, z) \mid$ assignment z satisfies as many clauses as possible in 3CNF formula $\phi\}$

promise problems *(very useful case between decision and search)*

for every input x, there is a set of acceptable outputs $y \in \{0,1\}$

partition into YES inputs, NO inputs, and DON'T-CARE inputs

Example

Max3SAT(α)

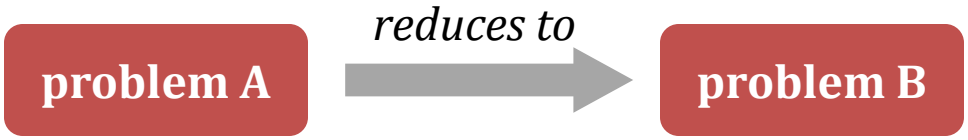
YES: satisfiable 3CNF formula

NO: at most α fraction of clauses satisfiable

resources

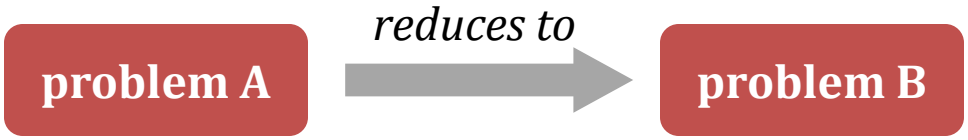
model	measure
Turing machine	time and space
	non-determinism, alternation
	randomness
	advice
circuits	size and depth
protocols	communication cost
resolution proofs	length
linear program	number of constraints (facets)

Reductions



Turing/Cook	solve A using an algorithm for B as a subroutine (use B as an oracle)
Karp	map instances of A to instances of B such that answer is preserved *(for decision & promise problems)*
Levin	map A-instances to B-instances such that any answer to B-instance can be pulled back *(for search problems)*

Reductions



Shows:

B is harder than A

meaningful if A is hard *(plausibly)*

Example: NP-hardness reductions

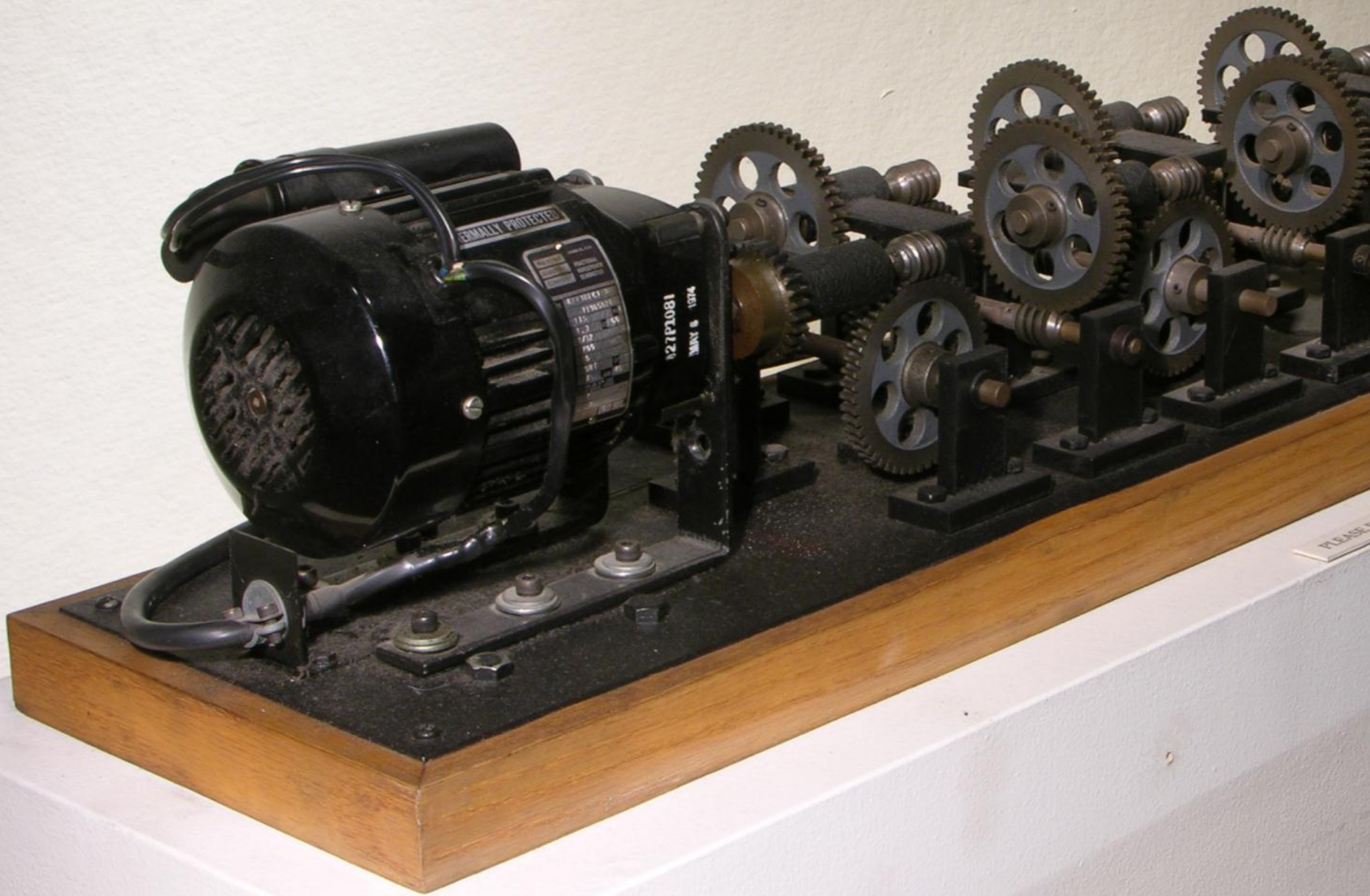
Warning:
3UNSAT is NP-hard w.r.t.
Cook reductions but *not* w.r.t.
Karp reductions (unless NP=coNP)

A is easier than B

meaningful if B is easy *(plausibly)*

Example: LP relaxations
(reduction to LP)

rounding algorithm is pull-back
from LP to original problem
(constructive Levin reduction)


THERMALLY PROTECTED

Machine with Concrete

Arthur Ganson