CS 4810: Homework 2

due 09/12 midnight

(your name + netid)

Collaborators: (names and netids)

Each problem is worth 20 points.

Problem 1

Show that a Boolean function can be represented by a straight-line program of length at most ℓ if and only if it can be represented by a Boolean circuit of size at most ℓ. (See the lecture notes for the definition of a straight-line program.)

Problem 2

Show that every finite subset of $\{0, 1\}^*$ is the language accepted by some DFA.

Problem 3

For reasons unknown, a man is travelling with a wolf, a goat and a very large cabbage. He is faced with unforeseen difficulties when he finds his path blocked by a roaring river, which may only be crossed by using a small boat tied to the shore. Indeed, the boat is so small that he can only possibly take one of his possessions (the cabbage is not to be taken lightly) on it at a time. Furthermore, his animal companions are proving less than cooperative and it is evident that neither the wolf and the goat nor the goat and the cabbage can be left ashore together without bipedal supervision, lest an undesirable *consumption event* occur.

Give a DFA that characterises the problem, with an appropriately chosen alphabet of possible actions for the traveller to take. By looking at your DFA or otherwise, find a sequence of actions that solves the conundrum.

Problem 4

Construct a DFA that accepts exactly the following language:

$L_{\text{odd}} = \{x \in \{0,1\}^* \mid x \text{ contains an odd number of 0's and an odd number of 1's}\}$.

Problem 5

(a) Construct a DFA for the following language:

$$L_7 = \{x \in \{0,1\}^* \mid x \text{ is the binary encoding of a multiple of } 7\}.$$

(b) Sketch a proof that, in fact, for any $m, k \in \mathbb{N}$, $k > 1$, the language

$$L_{m,k} = \{x \in \{0, 1, \ldots, k-1\}^* \mid x \text{ is the base-}k \text{ encoding of a multiple of } m\}$$

has a DFA.